

Resonance Scattering of Fast Electrons in a Single Crystal *

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It is shown that resonance scattering of fast ($pL \gg pR \gg 1$, p is the particle momentum, L is the longitudinal dimension of the potential, and R is its transverse dimension) charged particles occurs in the extended potential. The data on small-angle scattering of fast electrons in the crystal are interpreted on the basis of the examined effect.

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It is generally assumed that resonance scattering in elastic collisions applies to slow particles $pR \ll 1$ [1]. As will be shown below, however, resonances in the elastic scattering can also be present in fast particles, i.e., when $pR > 1$, if the particle is scattered by an extended, attracting potential. Physically, this effect consists of the following, the cross section of the extended, attracting potential has bound and quasi-bound states and hence the transverse component of the incident wave of a fast particle may undergo resonance scattering on these states. In this case the slow transverse motion of the particle is a necessary requirement, i.e., $pR \ll 1$. This imposes a constraint on the entrance angle of the particle $\theta_0 \ll 1/pR$.

Let us examine the scattering by a string potential which has the appearance of a square well (Fig. 1):

$$U(\vec{\rho}, z) = \begin{cases} -V_0, & \text{for } \rho \leq R, 0 \leq z \leq L, \\ 0, & \text{for } \rho > R, z < 0, z > L, \end{cases} \quad (1)$$

where L is the length of the string and $2R$ is its transverse diameter ($L \gg 2R$). If we assume that a fast particle $pL > pR > 1$ enters almost parallel to the z axis at an angle θ_0 , then from the condition for joining the wave function of a particle at the boundary $z = 0$ and $z = L$ and for the asymptotic behavior at infinity we obtain the scattering amplitude [2]

$$f(\theta, \phi) = (2) \frac{p}{2\pi i} \sum Q_{p\perp i}(\alpha, m) Q_{p\perp f}^*(\alpha, m) [\exp(i \frac{\alpha - p_{\perp}^2}{2p} L) - 1].$$

where

$$Q_{p\perp}(\alpha, m) = \int d^2 \vec{\rho} Z_{\alpha, m}(\vec{\rho}) \exp(ip_{\perp} \vec{\rho}).$$

is the amplitude of the transition from the state of the plane wave with the momentum p_{\perp} to the eigenstate $Z_{\alpha, m}(\vec{\rho})$ of the transverse motion in the potential (1); the summation is carried out over the compound states

with the transverse energy $\epsilon(n, m) = \alpha/2p$ and momentum m . To calculate the total cross section we use the optical theorem $\sigma = (4\pi/p) \text{Im}[f(\theta, \phi)]$. We obtain

$$\sigma = 4 \sum |Q_{p\perp i}(\alpha, m)|^2 \sin^2(\frac{\alpha - p_{\perp}^2}{2p} L). \quad (3)$$

It was shown earlier[3] that, as a result of scattering of a fast particle by a sufficiently extended potential ($L \gg pR^2$), the effective scattering angle becomes very small $\theta_{eff} \approx 1/\sqrt{L/p}$ and the corresponding effective impact parameter very large $\rho_{eff} \approx 1/p\theta_{eff} \approx \sqrt{L/p}$.

The particle with the azimuthal momentum m passes the string at a distance of $\approx m/p_{\perp}$ and hence is scattered by the potential (1) if this distance is smaller than the effective impact parameter

$$\frac{m}{p_{\perp i}} \leq \rho_{eff} = \sqrt{\frac{L}{p}}. \quad (4)$$

If

$$\frac{1}{p_{\perp i}} > \sqrt{\frac{L}{p}} \quad (\theta_0 < \frac{1}{\sqrt{pL}}),$$

then only the wave with $m = 0$ is scattered. In this case the scattering cross section (3), which is integrated and summed over the intermediate states, has the following form:

$$\sigma = \sigma_0 + \frac{2}{\pi} \sum \frac{|\epsilon^B(n, 0)|}{p} \frac{\sin^2[(|\epsilon^B(n, 0)| + \frac{p_{\perp}^2}{2p}) \frac{L}{2}]}{[|\epsilon^B(n, 0)| + \frac{p_{\perp}^2}{2p}]^2}. \quad (5)$$

where $\sigma_0 \approx (2/\pi)L/p$ is the scattering cross section from the continuous spectrum with $m = 0$, which was calculated elsewhere, [3] and the second term is determined by the bound states $[\epsilon^B(n, 0) < 0]$ in the transverse potential of the attracting string. We can easily see that the main contribution to the scattering cross section is introduced by bound states which are removed a distance of $\approx 1/L$ from the edge of the well. At the same time, the scattering cross section has a narrow peak $\approx 1/\sqrt{pL}$ in width near the zero angle of entry.

Let us assume now that the angle of entry is $1/\sqrt{pL} < \theta_0 < 1/pR$. The effective number of waves participating

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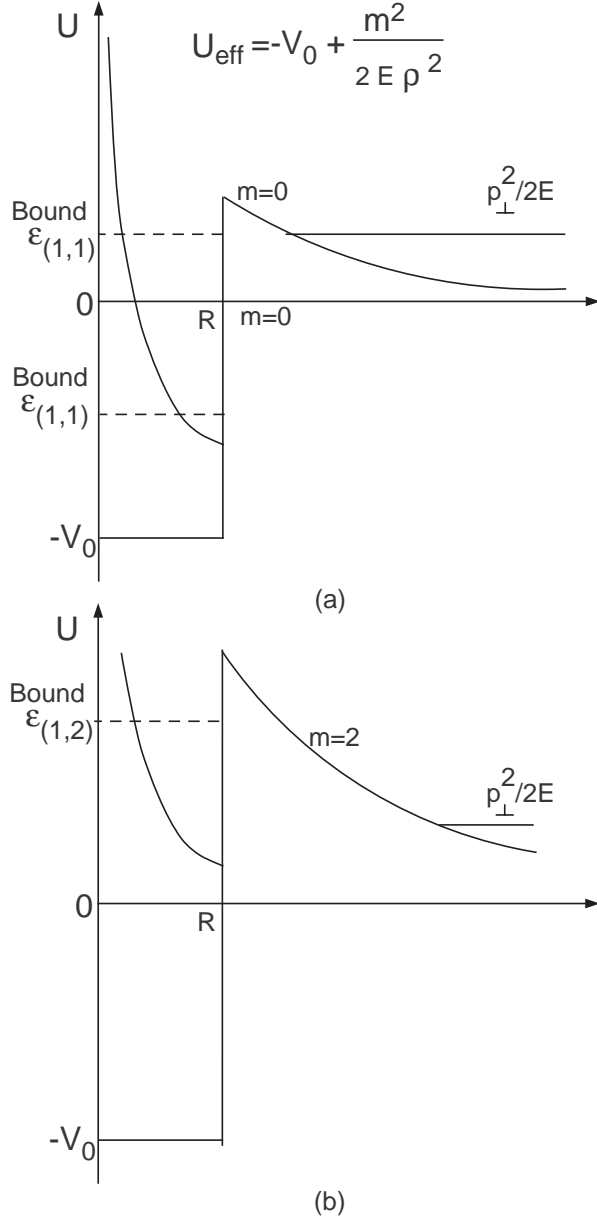


FIG. 1: Transverse cross section of the strings potential.

in the scattering in this case is determined by the inequality (4). It follows from relation (5) that the bound states [$\epsilon^B(n, m) < 0$] can be disregarded. However, for partial waves with $m > 0$ because of the presence of a centrifugal barrier $m^2/2ER^2$ quasi-bound states with $\epsilon^B(n, m)$ are developed in the effective potential (Fig. 1). Assuming that the barrier is infinitely high and substituting the square well for the effective well, we obtain

$$\epsilon^B(n, m) = \frac{\pi^2 n^2}{2ER^2} + \frac{m^2}{2ER^2} - V_0. \quad (6)$$

Taking into account the finite depth of the effective well ($\approx V_0$), we can calculate the number of quasi-bound

states corresponding to the azimuthal momentum m

$$n_{max}^2 = \frac{2EV_0R^2}{\pi^2}. \quad (7)$$

It can be seen that n_{max} , which is independent of m , is determined by the parameters of the well and by the total energy of the particle. In the energy region $E < \pi^2/2V_0R^2$ (for silicon $E < 20$ MeV) there is only one quasi-bound state $\epsilon^B(1, m)$ for a given m . It follows from Eq. (6) that $\epsilon^B(1, m) > 0$. Moreover, we always have $\epsilon(n, m) > \epsilon(n, m-1)$ (Fig. 1b). If the transverse energy now coincides with the energy of a certain quasibound state (Fig. 1a)

$$\frac{p_{\perp i}^2}{2E} = \frac{\pi^2 n^2 + m^2}{2ER^2} - V_0, \quad (8)$$

then we have resonance scattering. The scattering cross section, which can be determined from expression (3) by integrating over the continuous spectrum from 0 to ∞ , in this case has the Breit-Wigner form

$$\sigma = \sigma_0 + \frac{2L}{\pi p} \frac{\frac{1}{4}\Gamma^2}{(\theta_0 - \theta_{0res})^2 + \frac{1}{4}\Gamma^2}, \quad (9)$$

where σ_0 is the cross section for scattering at some distance from the resonance. We can easily obtain θ_{0res} from relation (8)

$$\theta_{0res} = \left(\frac{\pi^2 n^2 + m^2}{E^2 R^2} - 2 \frac{V_0}{E} \right)^{1/2}. \quad (10)$$

Determination of the finite centrifugal barrier using the standard quasiclassical treatment [1] yields the following angular width of the resonance

$$\Gamma \approx \theta_{0res} \cdot \exp[-2EV_0R^2 - \pi^2 n^2]^{1/2}. \quad (11)$$

If the angle of entry remains constant and the energy of the particle is varied, then from Eqs. (8), (9), and (11) we obtain

$$\begin{aligned} \sigma &= \sigma_0 + \frac{2L}{\pi p} \frac{\frac{1}{4}\bar{\Gamma}^2}{(E - E_{res})^2 + \frac{1}{4}\bar{\Gamma}^2}; \\ \bar{\Gamma} &\approx E_{res} \cdot \exp[-2E_{res}V_0R^2 - \pi^2 n^2]^{1/2}; \\ E_{res} &= \theta_0^{-2} \left[2V_0 + (4V_0^2 + 4\theta_0^2 R^{-2}(\pi^2 n^2 + m^2))^{1/2} \right] \end{aligned} \quad (12)$$

The indicated resonance can be observed in the crystal when the fast particle enters it at a small angle to the crystallographic axis.

In this case the longitudinal momentum transfer becomes very small $q_{\parallel} \approx 1/pR^2$ and the wave function of the particle, which is insensitive to the details of the behavior of the potential at approximately the interatomic distances, is determined by a certain potential averaged over the length of the chain[2]. Schiebel and Worm[4] observed the scattering of 15-MeV electrons along the

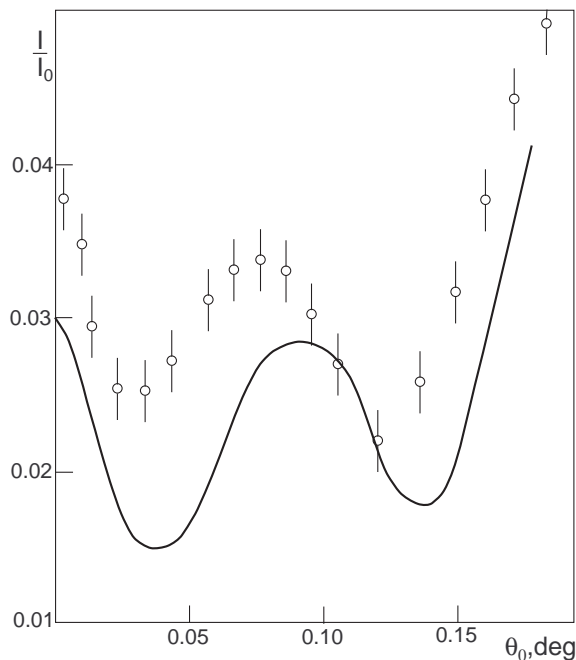


FIG. 2: The intensity of the 15-MeV electrons scattered in the $1.4 - \mu$ -thick silicon crystal as a function of the angle of entry θ_0 relative to the direction of the $\langle 111 \rangle$ axis, I_0 is the intensity of the incident particles. The experimental points were taken from Ref. 4; the solid curve denotes theoretical calculation.

$\langle 111 \rangle$ axis of a $1.4 - \mu\text{m}$ -thick *Si* crystal. Figure 2 shows a small-angle dependence of the intensity of the scattered particles on the angle of entry relative to the crystallographic $\langle 111 \rangle$ axis. This dependence cannot be explained in terms of the usual two-wave diffraction theory and the classical string scattering [5]. From the viewpoint of the discussion conducted above, the very narrow peak near the zero angle of entry is due to the true bound states; at the same time, its width $\approx 1/\sqrt{pL} \approx 0.02\text{degree}$ is very close to experimental result. Wide side peak is determined by scattering on the quasi-bound state with $n = 1$ and $m = 1$. From (10) and (11) for the parameters of *Si* $R^{-1} = m e^2 Z^{1/3} = 9.7 \cdot 10^3 \text{ eV}$, $V_0 = 23 \text{ eV}$, we obtain $\theta_{0\text{res}} \approx 0.1 \text{ degree}$ and $\Gamma \approx 0.5\theta_{0\text{res}}$ that is even more in line with the experimental results than would be expected from the evaluative expressions obtained.

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